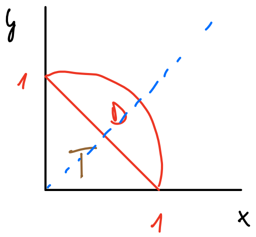
BARICENTRO INSIEME $D \subset \mathbb{R}^2$

ESEMPI



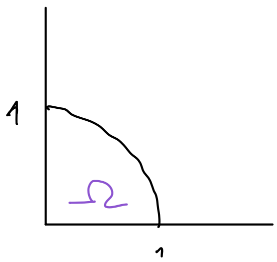
D = SEGMENTO CIRCOLARE O LUNETTA

• QUALE È IL BARICENTRO DI D? (x_c, y_c)

$$x_c = \frac{\iint_D x \, dx\, dy}{A(D)} \qquad y_D = \frac{\iint_D y \, dx\, dy}{A(D)}$$

$A(D)$ = AREA DI D

$$\overset{!}{=} \iint_D 1 \, dx\, dy$$



$D = \Omega \setminus T$

$$A(D) = \underset{\frac{1}{4}\pi}{\underset{\parallel}{A(\Omega)}} - \underset{\frac{1}{2}}{\underset{\parallel}{A(T)}}$$

DEVO CERCARE SIMMETRIA $\longrightarrow$ RISPETTO A BISETTRICE $y = x$

$$(x_c, y_c) \in D \iff (y_0, x_0) \in D$$

SO CHE $x_c = y_c$

$$\iint_D x \, dx\, dy = \underbrace{\iint_\Omega x \, dx\, dy}_{} - \underbrace{\iint_T x \, dx\, dy}_{}$$

• $\rho \in [0, 1] \quad \theta \in [0, \pi/2]$

$$\iint_\Omega = x \, dx\, dy = \int_0^1 \int_0^{\pi/2} \rho^2 \cos\theta \, d\rho \, d\theta$$

$$= \int_0^1 \rho^2 \, d\rho \cdot \int_0^{\pi/2} \cos\theta \, d\theta = 1/3$$

• SCELGO T – X NORMALE

$$x \in [0,1]$$

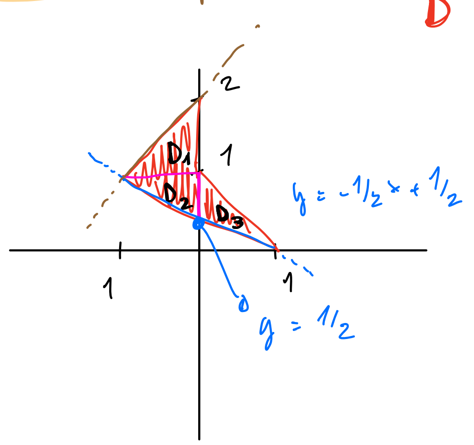
$$0 = g(x) \leq y \leq h(x) = 1 - x$$

$$= \int_0^1 \int_0^{1-x} x \, dy \, dx = 1/2 - 1/3$$

$$\iint_D x \, dx \, dy = 1/3 - 1/2 + 1/3 = 1/6$$

$$x_c = \frac{1/6}{\frac{\pi}{4} - 1/2} = y_c$$

ESEMPIO



DOMINIO

D È REGOLARE

$$x_c = \frac{\iint_D x \, dx \, dy}{A(D)}$$

$$D = D_1 \cup D_2 \cup D_3$$

$$A(D) = A(D_1) + A(D_2) + A(D_3)$$

$$\| \qquad \|$$

$$1/2 + 1/4 + 1/4 = 1$$

$$\iint_D = \iint_{D1} + \iint_{D_2} + \iint_{D_3}$$

• D_1 È X-NORMALE

$$x \in [-1, 0] \qquad 1 = g(x) \leq y \leq h(x) = x + 2$$

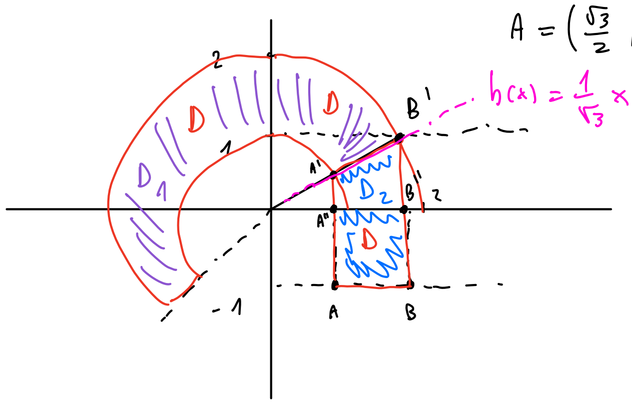
$$\int_{-1}^{0} \int_{1}^{x+2} x \, dy \, dx = \int_{-1}^{0} x(x+1) \, dx = \int_{-1}^{0} x^2 dx + \int_{-1}^{0} x \, dx$$

$$= \frac{1}{3}x^3 \Big|_{-1}^{0} + \frac{1}{2}x^2 \Big|_{-1}^{0} = \frac{1}{3} - \frac{1}{2} = -1/6 = x_c$$

- D_2 x - NORM.
- D_3 x - NORM.

ESERCIZIO

$A = \left(\frac{\sqrt{3}}{2}, -1\right)$ $B = \left(\sqrt{3}, -1\right)$



D REGOLARE

$$I = \iint_D xy \, dx\,dy$$

- $D1$: $\iint_{D_1} = \int_1^2 \int_{\pi/6}^{7/6\pi} \rho\cos\theta \, \rho\sin\theta \, \rho \, d\rho \, d\theta = \int_1^2 \rho^3 d\rho \cdot \int_{\pi/6}^{7/6\pi} \cos\theta \sin\theta \, d\theta = 1/2 \sin^2\theta$

$x = \rho\cos\theta$
$y = \rho\sin\theta$

$\rho \in [1, 2]$

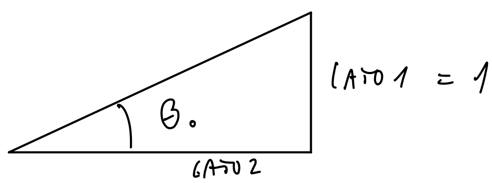
$\theta \in [\pi/6, \frac{7}{6}\pi]$

TRIANGOLO $A' O A''$ $A'' = \left(\frac{\sqrt{3}}{2}, 0\right)$

$A' = \left(\frac{\sqrt{3}}{2}, ?\right)$

$B' = (\sqrt{3}, 1)$ $B'' = (\sqrt{3}, 0)$ $\longrightarrow$

TRIANGOLO $OB''B'$



LATO 1 = LATO 2 $\cdot$ TAN(θ_0)

$1 = \sqrt{3} \cdot \tan(\theta_0)$

$\Rightarrow \tan(\theta_0) = 1/\sqrt{3}$

$\Downarrow$

$\pi/6$

• D_2 : x - NORM.

$x \in \left[\frac{\sqrt{3}}{2}, \sqrt{3}\right]$

$-1 = g(x) \le y \le h(x) = \frac{1}{\sqrt{3}} x$

$$\iint_{D_2} = \int_{\sqrt{3}/2}^{\sqrt{3}} \int_{-1}^{1/\sqrt{3}x} xy \, dy \, dx$$

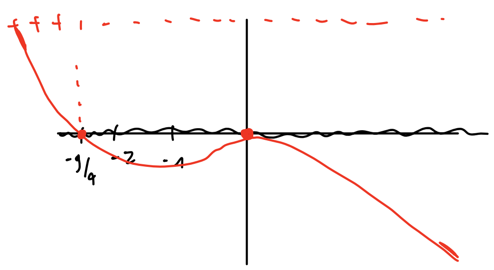
ESEMPIO

$f(x,y) = -4x^3 + 2x^2 y - x y^2 - 9x^2$

$D = \mathbb{R}^2$

a) ABBOZZARE SEZIONE DI f PER $y=0$

$\varphi(x) = f(x,0) = -4x^3 - 9x^2$ SOTTODOMINIO ASSE x



P.TI STAZIO.:

$f_x = -12x^2 + 4xy - y^2 - 18x$

$f_y = 2x^2 - 2xy$

$= -x^2(4x+9)$

$4x+9<0 \quad 4x<-9 \quad x<-9/4$

PER $x < -9/4$ È POSITIVO

$$\begin{cases} -12x^2 + 4xy - y^2 - 18x \\ x(x-y) = 0 \end{cases} \quad x = y \ (1), \quad x = 0 \ (2)$$

P.TI STAZIO. : $= \{(0,0), (-2,-2)\}$

2° PASSO HESSIANA

$$H = \begin{pmatrix} -24x + 4y - 18 & 4x - 2y \\ 4x - 2y & -2x \end{pmatrix}$$

$\det(H) = 22 > 0$

A = P.TO MINIMO

$$H(0,0) = \begin{pmatrix} -18 & 0 \\ 0 & 0 \end{pmatrix} = |H(0,0)| = 0$$

SEMIDEFINITA → BOH ?!?

$\Delta f(0,0) = f(h,k) - f(0,0)$ $\qquad h,k \to 0$

$f(0,0) = 0$

$-4h^3 + 2h^2k - hk^2 - 9h^2$

$f(h,0) = -4h^3 - 9h^2$

$= -h^2(-4h - 9)$

pos

$h > 4/9$

$-4h - 9 < 0 \Rightarrow -4h < 9 \qquad 4h > 9$

QUINDI PER $h \to 0 \to$ NEGATIVO

$h < 4/9$

HINT

$x = -\frac{1}{9} y^2$ $\Big\}$ VEDI ES. 9

FOGLIO 9 SOLUZIONI

b) DIRE QUAL È IL VERSORE DI MASSIMA DISCESA NEL PUNTO $x_0 y_0 = (1,1)$

$\nabla f(1,1) \to$ VETT. DI MAX SALITA

$= (f_x(1,1), f_y(1,1)) = [-27 \; , \; 0]$

VERSORE: $\sqrt{(-27)^2 + 0^2} = 27 \Rightarrow \frac{(-27,0)}{27} = (-1,0)$ VERSORE MAX SALITA

$\Downarrow$

MAX DISCESA È: $(1,0)$